National
Qualifications
2024

2024 Mathematics Paper 1 Non-calculator

Advanced Higher

Question Paper Finalised Marking Instructions

Marking Instructions for each question

Question			Generic scheme	Illustrative scheme	Max mark
1.	(a)		•[1] begin differentiation [1] •[2] apply chain rule [2]	•[1] $-\operatorname{cosec}^2 3x$ •[2] $-3\operatorname{cosec}^2 3x$	**2**

Notes:

1. Where a candidate equates y to the derivative, •[1] is still available.
2. At •[2], accept $-\operatorname{cosec}^2 3x \times 3$.

Commonly Observed Responses:

Where a candidate writes $\cot 3x$ as $\dfrac{\cos 3x}{\sin 3x}$:

$\dfrac{-3\sin 3x \sin 3x - \ldots}{\sin^2 3x}$ or $\dfrac{\ldots - 3\cos 3x \cos 3x}{\sin^2 3x}$ award •[1]

$\dfrac{-3}{\sin^2 3x}$ award •[2]

Question			Generic scheme	Illustrative scheme	Max mark
	(b)		•[3] evidence of use of product rule with one term correct [1,2] •[4] complete differentiation	•[3] $5(4x-7)^{\frac{1}{2}} + \ldots$ or $\ldots + 5x \times \frac{1}{2} \times 4(4x-7)^{-\frac{1}{2}}$ •[4] $5(4x-7)^{\frac{1}{2}} + 10x(4x-7)^{-\frac{1}{2}}$	**2**

Notes:

1. Except where it results from rearrangement of a correct answer, if a candidate produces one term only, award 0/2.
2. Where a candidate equates the derivative to the original function, •[3] is not available (see COR).

Commonly Observed Responses:

Candidate equates derivative to original function:

$$f(x) = 5x(4x-7)^{\frac{1}{2}}$$
$$= 5(4x-7)^{\frac{1}{2}} + 10x(4x-7)^{-\frac{1}{2}}$$

Do not award •[3].

Question			Generic scheme	Illustrative scheme	Max mark
2.	(a)		•¹ calculate modulus or argument [1,2]	•¹ $\sqrt{2}$ or $\frac{\pi}{4}$	**2**
			•² write in polar form [1,2]	•² $\sqrt{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)$	

Notes:

1. Accept arguments expressed in degrees provided a degree symbol appears at least once in part (a) or (b); otherwise withhold •².

2. Any working leading to calculation of the argument (and the modulus) must be consistent (General Marking Principle (n)).

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
	(b)		•³ apply de Moivre's theorem to argument [1,2,3]	•³ $\cos\frac{8\pi}{4}+i\sin\frac{8\pi}{4}$	**2**
			•⁴ complete process [4,5]	•⁴ 16	

Notes:

1. For the award of •³, there must be a single argument. Disregard the form of the modulus.
2. It is not sufficient at •³ for the argument to be expressed as a variable.
3. Where a candidate has produced a zero argument at •², •³ is available only where 0 is explicitly multiplied by 8.
4. Where a candidate has produced a modulus for z equal to ±1, •⁴ is not available.
5. At •⁴, do not accept $\ldots(\cos 2\pi + i\sin 2\pi)$.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
3.	(a)		•¹ interpret geometric sequence [1,2]	•¹ $ar^2 = 36$ and $ar^4 = 16$	**2**
			•² calculate common ratio [2,3]	•² $\frac{2}{3}$	

Notes:

1. Award •¹ for $r^2 = \frac{16}{36}$
2. For the award of •¹, there must be some evidence of strategy, eg 36 24 16. For a statement of the answer without justification, award •² only.
3. There is no requirement for an explicit rejection of a negative answer at •².

Commonly Observed Responses:

$r = \frac{16}{36} = \sqrt{\frac{16}{36}} = \frac{4}{6} = \frac{2}{3}$ award •² only

Question			Generic scheme	Illustrative scheme	Max mark
	(b)		•³ calculate first term	•³ 81	**1**

Notes:

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
	(c)		•⁴ show condition is satisfied [1,2,3]	•⁴ $\left\lvert\frac{2}{3}\right\rvert < 1$ or equivalent	**1**

Notes:

1. For •⁴ $\frac{2}{3}$ may be replaced with a letter consistent with their answer to (a). However, in the case where a candidate obtains a value in (a) outside the open interval $(-1,1)$, •⁴ is available only where they also acknowledge that there is no sum to infinity.
2. For •⁴, accept an equivalent statement in words. However, if a candidate uses the term "between", it must be explicitly stated that it is strictly between.
3. Where the answer contains incorrect (rather than insufficient) information (before, between or after correct information), •⁴ is not available.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
3.	(d)		$\bullet^5$ calculate sum to infinity [1]	$\bullet^5$ 243	**1**

Notes:

1. Where an incorrect value is calculated in (a), $\bullet^5$ is available only where that value satisfies the condition for convergence.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
4.	(a)		•¹ find determinant or adjunct [1,2] •² find A^{-1} [1,2]	•¹ $\det A = 7$ or $\begin{pmatrix} 3 & -1 \\ -11 & 6 \end{pmatrix}$ •² $\frac{1}{7}\begin{pmatrix} 3 & -1 \\ -11 & 6 \end{pmatrix}$	**2**

Notes:

1. For correct answer with no working, award 2/2.
2. Where the determinant has not been explicitly identified, •¹ may be awarded for $\frac{1}{7}\begin{pmatrix} \cdots & \cdots \\ \cdots & \cdots \end{pmatrix}$.

Commonly Observed Responses:

	(b)		•³ determine M in terms of A^{-1} and B [1] •⁴ find matrix M [2,3]	•³ $M = A^{-1}B$ •⁴ $\begin{pmatrix} -1 & 1 \\ 2 & -3 \end{pmatrix}$	**2**

Notes:

1. Where a candidate has written $\frac{B}{A}$, award •³ only if they subsequently write or calculate $A^{-1}B$.
2. At •⁴, accept $\frac{1}{7}\begin{pmatrix} -7 & 7 \\ 14 & -21 \end{pmatrix}$
3. At •⁴, the only acceptable multiplications are $A^{-1}B$ or BA^{-1}.

Commonly Observed Responses:

COR A

For $BA^{-1} = \frac{1}{7}\begin{pmatrix} -45 & 22 \\ -37 & 17 \end{pmatrix}$, award •⁴.

COR B

For candidates who use simultaneous equations:

Award •³ for four equations, or a pair of equations with solutions, eg

$$\begin{aligned} 6a + c &= -4 \\ 11a + 3c &= -5 \\ 6b + d &= 3 \\ 11b + 3d &= 2 \end{aligned}$$

or eg $\begin{aligned} 6a + c &= -4 \\ 11a + 3c &= -5 \end{aligned}$ leading to $a = -1,\ c = 2$

Question			Generic scheme	Illustrative scheme	Max mark
5.	(a)		•[1] expression for $f(-x)$ [1,2] •[2] justify that the function is odd [1,2]	•[1] $f(-x)=(-x)^3-(-x)$, stated or implied •[2] $-x^3+x8$ leading to $=-f(x)$ or $-(x^3-x)$ odd	2

Notes:

1. Where a candidate has used an exclusively graphical approach, a sketch including roots and stationary points is required for the award of •[1]. For •[2], reference must made to half-turn symmetry about the origin.
2. Award 0/2 for a numerical approach.

Commonly Observed Responses:

	(b)		•[3] equate second derivative to 0 [1,2,3] •[4] consider sign of $f''(x)$ for $x<0$ and $x>0$ and state conclusion [4]	•[3] $6x=0$ •[4] $x>0 \Rightarrow f''(x)>0$ and $x<0 \Rightarrow f''(x)<0$ $\therefore$ POI	2

Notes:

1. Given that the second derivative exists for all x, it is sufficient to consider only a zero second derivative.
2. Do not withhold •[3] where a candidate states that points of inflection occur when the second derivative equals zero.
3. Where a candidate does not explicitly equate $6x$ to zero, •[3] may be awarded for $f''(x)=0$ provided they also write $f''(x)=6x$.
4. May be awarded where $f''(x)$ and $f''(-x)$ have been calculated for a specific value of x close to 0 and shown to have opposite signs.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
6.	(a)		•1 obtain matrix	•1 $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	**1**

Notes:

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
	(b)		•2 describe effect [1]	•2 reflection in the line $y = x$.	**1**

Notes:

1. Reference to reflection (in/across/along) $y = x$ must appear.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
	(c)		•3 correct order for multiplication [1]	•3 $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	**2**
			•4 complete multiplication [2,3]	•4 $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	

Notes:

1. Do not withhold •3 for incorrect information prior to $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$.
2. •4 is not available if a candidate incorrectly identified $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ at •1.
3. **Beware** of incorrect working leading to $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ at •4.

Commonly Observed Responses:

Incorrect order of multiplication:

$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

Question			Generic scheme	Illustrative scheme	Max mark
7.	(a)		•[1] apply product rule [1]	•[1] $2xy + x^2\frac{dy}{dx}$ or $4y^2 + 8xy\frac{dy}{dx}$	**3**
			•[2] complete differentiation [1]	•[2] $2xy + x^2\frac{dy}{dx} + 4y^2 + 8xy\frac{dy}{dx} = 0$	
			•[3] find expression for $\frac{dy}{dx}$ [2,3]	•[3] $\frac{dy}{dx} = \frac{-2xy - 4y^2}{x^2 + 8xy}$	

Notes:

1. Terms need not be simplified at •[1] or •[2].
2. Award •[3] only where $\frac{dy}{dx}$ appears more than once after the candidate has completed their differentiation.
3. Withhold •[3] if there is further incorrect simplification of $\frac{dy}{dx}$.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
	(b)		•[4] equate expression for $\frac{dy}{dx}$ to 0 [1,2,3]	•[4] $\frac{-2xy - 4y^2}{x^2 + 8xy} = 0$	**3**
			•[5] state linear relationship between y and x at stationary point [2,4,5,6]	•[5] eg $y = \frac{-x}{2}$, $x = -2y$, $x + 2y = 0$	
			•[6] determine coordinates of stationary point [2,5,6,7]	•[6] $(-4, 2)$	

Notes:

1. Award •[4] for substitution of $\frac{dy}{dx} = 0$ into the equation at •[2].
2. Where a candidate has failed to differentiate the RHS of the original equation, only •[4] is available.
3. At •[4], accept $-2xy - 4y^2 = 0$.
4. For the award of •[5], the relationship need not be simplified.
5. Where a candidate equates the denominator to zero, •[5] and •[6] are not available regardless of the processing of the numerator.
6. Disregard the appearance of $y = 0$.
7. For the award of •[6], there must be a linear relationship between y and x at •[5].

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
8.			•[1] differentiate	•[1] $du = 2\sec^2 2x dx$ or $\frac{du}{dx} = 2\sec^2 2x$	4
			•[2] determine new limits and begin to rewrite integrand [1,2]	•[2] $\int_0^1 \ldots du$	
			•[3] complete integrand [2,3,4,5]	•[3] $\int_0^1 \frac{1}{2}\sqrt{u}\, du$	
			•[4] evaluate [5]	•[4] $\frac{1}{3}$	

Notes:

1. Any working leading to calculation of new limits at •[2] must be consistent (General Marking Principle (n)).
2. Except as indicated in COR A, candidates must produce an integral including the correct new limits and du at some point for the award of •[2].
3. Except as indicated in COR A, disregard the omission of du and/or limits for the award of •[3].
4. Where the integrand contains terms in x, •[3] is still available provided these terms are clearly and correctly "cancelled out".
5. Where candidates attempt to integrate an expression containing both u and x, where x is either inside the integrand or taken outside as a constant, •[3] and •[4] are not available.

Question		Generic scheme	Illustrative scheme	Max mark
8.	(continued)			

Commonly Observed Responses:

COR A

No limits in new integral and return to original variable:

$\int \frac{1}{2}\sqrt{u}\,du$ — award $\bullet^3$ provided du appears at this stage

$\left[\frac{1}{3}(\tan 2x)^{\frac{3}{2}}\right]_0^{\frac{\pi}{8}}$ — award $\bullet^2$

COR B

Wrong limits in new integral and return to original variable:

$\int_0^{\frac{\pi}{8}} \ldots du$ — do not award $\bullet^2$

$\int_0^{\frac{\pi}{8}} \frac{1}{2}\sqrt{u}\,du$ — award $\bullet^3$

$\left[\frac{1}{3}(\tan 2x)^{\frac{3}{2}}\right]_0^{\frac{\pi}{8}}$ leading to $\frac{1}{3}$ — award $\bullet^4$

COR C

Wrong limits in new integral and new limits used in evaluation:

$\int_0^{\frac{\pi}{8}} \ldots du$ — do not award $\bullet^2$

$\int_0^{\frac{\pi}{8}} \frac{1}{2}\sqrt{u}\,du$ — award $\bullet^3$

$\left[\frac{1}{3}u^{\frac{3}{2}}\right]_0^1$ leading to $\frac{1}{3}$ — award $\bullet^4$

COR D

Wrong limits in new integral and no return to original variable:

$\int_0^{\frac{\pi}{8}} \ldots du$ — do not award $\bullet^2$

$\int_0^{\frac{\pi}{8}} \frac{1}{2}\sqrt{u}\,du$ — award $\bullet^3$

$\left[\frac{1}{3}u^{\frac{3}{2}}\right]_0^{\frac{\pi}{8}}$ — do not award $\bullet^4$

[END OF MARKING INSTRUCTIONS]

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Qualifications
2024

2024 Mathematics Paper 2

Advanced Higher

Question Paper Finalised Marking Instructions

Marking Instructions for each question

Question			Generic scheme	Illustrative scheme	Max mark
1.			•[1] evidence of use of quotient rule with denominator and one term correct in the numerator [1,2]	•[1] $\dfrac{7\cos 7x\left(1+x^2\right)-\ldots}{\left(1+x^2\right)^2}$ **OR** $\dfrac{\ldots-2x\sin 7x}{\left(1+x^2\right)^2}$	2
			•[2] complete differentiation [2]	•[2] $\dfrac{7\cos 7x\left(1+x^2\right)-2x\sin 7x}{\left(1+x^2\right)^2}$	

Notes:

1. Where a candidate equates y to the derivative, •[1] is still available.
2. For candidates who use the product rule, no marks may be awarded where an attempt to differentiate $\dfrac{1}{1+x^2}$ produces an inverse trigonometric function.

Commonly Observed Responses:

COR A
Product Rule

$7\cos 7x\left(1+x^2\right)^{-1}-\ldots$ or $\ldots-2x\sin 7x\left(1+x^2\right)^{-2}$ award •[1]

$7\cos 7x\left(1+x^2\right)^{-1}-2x\sin 7x\left(1+x^2\right)^{-2}$ award •[2]

COR B
Logarithmic Differentiation

$\ln y = \ln\sin 7x \quad \ln\left(1 \quad x^2\right)$ AND $\dfrac{1}{y}\dfrac{dy}{dx}=\ldots$ award •[1]

$\dfrac{dy}{dx}=\dfrac{\sin 7x}{1+x^2}\left(\dfrac{7\cos 7x}{\sin 7x}-\dfrac{2x}{1+x^2}\right)$ award •[2]

Question			Generic scheme	Illustrative scheme	Max mark
2.			•[1] complete algorithm [1]	•[1] $533 = 455\times1+78$ $455 = 78\times5+65$ $78 = 65\times1+13$ $(65 = 13\times5)$	**3**
			•[2] express gcd in terms of 455 and 533	•[2] eg $13 = (533-455\times1)\times6-455$	
			•[3] obtain a and b [2,3]	•[3] $a=6$, $b=-7$	

Notes:

1. At •[1] the gcd and the final line of working do not have to be stated explicitly.
2. The minimum requirement for •[3] is $13 = 533\times6+455\times(-7)$.
3. Do not accept $13 = 6\times533-7\times455$ where the values of a and b have not been explicitly stated.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
3.	(a)		•[1] set up augmented matrix [1]	•[1] $\left(\begin{array}{ccc|c} 1 & -1 & -3 & 1 \\ 2 & -3 & -5 & 8 \\ 1 & 2 & \lambda & -7 \end{array}\right)$	**4**
			•[2] obtain two zeros [2]	•[2] $\left(\begin{array}{ccc|c} 1 & -1 & -3 & 1 \\ 0 & 1 & -1 & -6 \\ 0 & 3 & \lambda+3 & -8 \end{array}\right)$	
			•[3] complete row operations [2]	•[3] $\left(\begin{array}{ccc|c} 1 & -1 & -3 & 1 \\ 0 & 1 & -1 & -6 \\ 0 & 0 & \lambda+6 & 10 \end{array}\right)$	
			•[4] obtain expression for z [3]	•[4] $z = \dfrac{10}{\lambda+6}$	

Notes:

1. Where a candidate equates a 3×3 matrix to a 3×1 matrix, •[1] is not available. Otherwise, accept eg $x, y, z, =$ left in.
2. Only Gaussian elimination (ie a systematic approach using EROs) is acceptable for the award of •[2] and •[3].
3. Do not accept an answer of $(\lambda+6)z = 10$ when awarding •[4].

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
	(b)		•[5] state value of λ [1]	•[5] -6	**1**

Notes:

1. Do not award •[5] for $z = -6$.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
	(c)		•[6] find solution [1]	•[6] $x = -3,\ y = \ 4,\ z \ \ 2$	**1**

Notes:

1. Where a candidate has made an error in (a), the value of x may be obtained from either the original equations or their final augmented matrix.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
4.			•[1] state auxiliary equation [1]	•[1] $m^2 - 2m - 8 = 0$	5
			•[2] state general solution [2]	•[2] $y = Ae^{-2x} \quad Be^{4x}$	
			•[3] differentiate	•[3] $\frac{dy}{dx} = 2Ae^{-2x} \quad 4Be^{4x}$	
			•[4] solve for one constant	•[4] $A = -5$ or $B = 3$	
			•[5] find second constant and state particular solution [2]	•[5] $y = 5e^{-2x} \quad 3e^{4x}$	

Notes:

1. •[1] is not available where $'=0'$ has been omitted.
2. Disregard the omission of $'y = \ldots'$ at •[2].
3. Where a candidate does not give an expression for the derivative, •[3] may be awarded for $-2A + 4B = 22$.
4. Do not award •[5] if $'y = \ldots'$ does not appear at that stage.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
5.	(a)		•[1] state general term [1,2,3,4]	•[1] $\binom{16}{r}\left(2x^2\right)^{16-r}\left(-\frac{1}{x^3}\right)^r$	**3**
			•[2] simplify powers of x	•[2] $2^{16-r}(-1)^r$	
			OR	**OR**	
			•[2] coefficients and signs [5]	•[2] x^{32-5r}	
			•[3] complete simplification [5,6,7,8]	•[3] $\binom{16}{r}(-1)^r\,2^{16-r}\,x^{32-5r}$	

Notes:

1. Candidates may also proceed from $\binom{16}{r}\left(2x^2\right)^r\left(-\frac{1}{x^3}\right)^{16-r}$, leading to $\binom{16}{r}(-1)^{16-r}\,2^r\,x^{5r-48}$.
2. Where a candidate writes out a full expansion, •[1], •[2] and •[3] are not available, unless the general term is identifiable in (b).
3. Where a candidate omits $\binom{16}{r}$, do not award •[1].
4. Where a candidate does not fully substitute for n at first, •[1] is available only where a correct expression in terms of x and r appears at a later stage.
5. Where a candidate produces a numerical power of 2 or (-1), this must be evaluated for the award of the coefficient mark in the general term.
6. Award full marks if the expression at •[3] appears without working.
7. Where a candidate in (a) produces an incorrect further simplification subsequent to the correct answer (eg $2^{16-r}(-1)^r$ becomes $(-2)^{16}$), •[3] is not available.
8. Note that $\binom{16}{r}(-2)^{16-r}\,x^{32-5r}$ is a correct simplification in this case.

Question			Generic scheme	Illustrative scheme	Max mark
5.	(a)		**(continued)**		

Commonly Observed Responses:

COR A

General term has not been isolated

$$\sum_{r=0}^{16}\binom{16}{r}\left(2x^2\right)^{16-r}\left(-\frac{1}{x^3}\right)^r$$

$$=\sum_{r=0}^{16}\binom{16}{r}(\ \ 1)^r\,2^{16-r}\,x^{32-5r}$$

Do not award •¹. Award •² and •³.

COR B

General term has been isolated

$$\sum_{r=0}^{16}\binom{16}{r}\left(2x^2\right)^{16-r}\left(-\frac{1}{x^3}\right)^r$$

$$=\binom{16}{r}(\ \ 1)^r\,2^{16-r}\,x^{32-5r}$$

Disregard the incorrect use of the final equals sign. Award •¹, •² and •³.

COR C

Binomial expression has been equated to the general term

$$\left(2x^2-\frac{1}{x^3}\right)^{16}=\binom{16}{r}(-1)^r\,2^{16-r}\,x^{32-5r}$$

Disregard the incorrect use of the equals sign. Award •¹.

COR D

Negative sign omitted

$$\binom{16}{r}\left(2x^2\right)^{16-r}\left(\frac{1}{x^3}\right)^r$$

Do not award •¹ or •³, but •² is still available.

COR E

Brackets omitted around -1 in final expression

$$\binom{16}{r}-1^r\,2^{16-r}\,x^{32-5r}$$

Do not award •³.

COR F

Negative sign has been associated with x in final expression

$$\binom{16}{r}2^{16-r}\left(-x^{32-5r}\right)\text{ or }\binom{16}{r}2^{16-r}(-x)^{32-5r}$$

Award •² for the appearance of x^{32-5r}.
Do not award •³.

Question			Generic scheme	Illustrative scheme	Max mark
5.	(b)		$\bullet^4$ determine the value of r [1,2] $\bullet^5$ evaluate coefficient [2,3,4]	$\bullet^4$ $r = 10$ $\bullet^5$ 512512	**2**

Notes:

1. A candidate starting from $\binom{16}{r}\left(2x^2\right)^r\left(-\dfrac{1}{x^3}\right)^{16-r}$ should have $r = 6$ at $\bullet^4$.
2. Where a candidate writes out a full expansion, $\bullet^4$ may be awarded only if the expansion is complete and correct at least as far as the required term (in either direction). The required term must clearly identified in the expansion for $\bullet^5$ to be awarded.
3. Where a candidate has omitted $\binom{16}{r}$ in (a), do not award $\bullet^5$, unless it now appears in (b).
4. At $\bullet^5$ accept $\dfrac{512512}{x^{18}}$.

Commonly Observed Responses:

Binomial expansion

$$65536x^{32} - 524288x^{27} + 1966080x^{22} - 4587520x^{17} + 7454720x^{12} - 8945664x^{7} + 8200192x^{2}$$
$$-5857280x^{-3} + 3294720x^{-8} - 1464320x^{-13} + 512512x^{-18} - 139776x^{-23} + 29120x^{-28}$$
$$-4480x^{-33} + 480x^{-38} - 32x^{-43} + x^{-48}$$

Question			Generic scheme	Illustrative scheme	Max mark
6.	(a)		•1 begin to find $\frac{dy}{dt}$	•1 $\frac{dy}{dt} = 4\ln t \quad \ldots$ **OR** $\frac{dy}{dt} = \ldots \quad 4t + \times \frac{1}{t}$	3
			•2 find $\frac{dy}{dt}$ 1	•2 $\frac{dy}{dt} = 4\ln t \quad 4$	
			•3 simplified expression for $\frac{dy}{dx}$ 2	•3 $\frac{dy}{dx} = \frac{2(\ln t + 1)}{t}$	

Notes:

1. Accept an unsimplified expression for $\frac{dy}{dt}$ for the award of •2.
2. Accept $\frac{dy}{dx} = \frac{2\ln t + 2}{t}$.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
	(b)		•4 begin to differentiate $\frac{dy}{dx}$ with respect to t 1	•4 $\frac{\frac{2}{t}t - \ldots}{t^2}$ or $\frac{\ldots - 2(\ln t + 1)}{t^2}$	3
			•5 complete differentiation of $\frac{dy}{dx}$ with respect to t 1,2	•5 $\frac{\frac{2}{t}t - 2(\ln t + 1)}{t^2}$	
			•6 simplify $\frac{d^2y}{dx^2}$ 1	•6 $\frac{-\ln t}{t^3}$	

Notes:

1. Where a candidate has not attempted to differentiate their answer to (a) with respect to t, award 0/3.
2. Where a candidate produces an incorrect expression in (a), differentiation must involve a product or quotient, including a logarithmic term, for the award of •5.

Question			Generic scheme	Illustrative scheme	Max mark
6.	(b)		(continued)		

Commonly Observed Responses:

Candidate uses a formula method

COR A (starting from unsimplified first derivative)

$\dfrac{\frac{4}{t}\times 2t-\ldots}{(2t)^3}$ or $\dfrac{\ldots-2(4\ln t+4)}{(2t)^3}$ award $\bullet^4$

$\dfrac{\frac{4}{t}\times 2t-2(4\ln t+4)}{(2t)^3}$ award $\bullet^5$

$\dfrac{-\ln t}{t^3}$ award $\bullet^6$

COR B (starting from simplified first derivative)

$\dfrac{\frac{2}{t}\times t-\ldots}{t^3}$ or $\dfrac{\ldots-(2\ln t+2)}{t^3}$ award $\bullet^4$

$\dfrac{\frac{2}{t}\times t-(2\ln t+2)}{t^3}$ award $\bullet^5$

$\dfrac{-2\ln t}{t^3}\times\dfrac{1}{2}$ leading to $\dfrac{-\ln t}{t^3}$ award $\bullet^6$

Question			Generic scheme	Illustrative scheme	Max mark
7.	(a)	(i)	Method 1	Method 1	**2**
			•1 all three derivatives and all four evaluations	•1 $f(x)=e^{2x}$ $f(0)=1$ $f'(x)=2e^{2x}$ $f'(0)=2$ $f''(x)=4e^{2x}$ $f''(0)=4$ $f'''(x)=8e^{2x}$ $f'''(0)=8$ stated or implied	
			•2 obtain simplified expression [1]	•2 $1+2x+2x^2+\frac{4}{3}x^3$	
			Method 2	Method 2	
			•1 write down Maclaurin series for e^x	•1 $1+x+\frac{x^2}{2!}+\frac{x^3}{3!}$ stated or implied	
			•2 substitute and simplify [1]	•2 $1+2x+2x^2+\frac{4}{3}x^3$	

Notes:

1. Evidence of full simplification may appear in (b).

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
7.	(a)	(ii)	Method 1 •³ all three derivatives and all four evaluations [1] •⁴ obtain simplified expression [2,3] Method 2 •³ write down Maclaurin series for $\sin x$ [1] •⁴ substitute and simplify [2,3]	Method 1 •³ $g(x)=\sin 3x$ $g(0)=0$ $g'(x)=3\cos 3x$ $g'(0)=3$ $g''(x)=-9\sin 3x$ $g''(0)=0$ $g'''(x)=-27\cos 3x$ $g'''(0)=-27$ stated or implied •⁴ $3x-\frac{9}{2}x^3$ Method 2 •³ $x-\frac{x^3}{3!}$ stated or implied •⁴ $3x-\frac{9}{2}x^3$	**2**

Notes:

1. Disregard the repeated use of notation from (a) eg $f(x)$.
2. Do not accept $3x+-\frac{9}{2}x^3$ unless resolved in (b), but accept $3x+\frac{-9}{2}x^3$.
3. Evidence of full simplification may appear in (b).

Commonly Observed Responses:

	(b)		•⁵ set up composition of expansions [1,2] •⁶ expand and simplify [1,2,3]	•⁵ $1+2\left(3x-\frac{9}{2}x^3\right)+2\left(3x-\frac{9}{2}x^3\right)^2+\frac{4}{3}\left(3x-\frac{9}{2}x^3\right)^3$ •⁶ $1+6x+18x^2+27x^3$	**2**

Notes:

1. For candidates who attempt to multiply expressions from (a), award 0/2.
2. For candidates who attempt an answer from first principles, award 0/2.
3. Disregard higher order terms, whether correct, incorrect or absent.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
8.			•1 correct form of integral including limits 1	•1 $\int_0^a \pi y^2 dx$	5
			•2 square y and substitute 1	•2 $\int_0^a \pi \frac{1}{1+x^2} dx$	
			•3 integrate 2	•3 $\tan^{-1} x$	
			•4 substitute limits and simplify 3	•4 $\tan^{-1} a = \frac{\pi}{3}$	
			•5 evaluate 3	•5 $\sqrt{3}$	

Notes:

1. For the award of •1:
    a. limits must appear at some point
    b. dx must appear at some point.
2. For the award of •3, the integration must be beyond Higher level.
3. Neither •4 nor •5 is available where a candidate produces either:
    a. limits which are both constants
    b. an expression which is not an inverse trigonometric function.

Commonly Observed Responses:

Candidates who use a as their lower limit:

$\int_a^0 \pi y^2 dx$	award •1
$\int_a^0 \pi \frac{1}{1+x^2} dx$	award •2
$\tan^{-1} x$	award •3
$-\tan^{-1} a = \frac{\pi}{3}$	award •4
$-\sqrt{3}$	award •5

Question			Generic scheme	Illustrative scheme	Max mark
9.	(a)		•¹ state expression	•¹ $-3+2d$	**1**

Notes:

Commonly Observed Responses:

	(b)		•² find common difference	•² 4	**1**

Notes:

Commonly Observed Responses:

	(c)		•³ substitute [1,2,3] •⁴ write quadratic in standard form and solve [1,4,5] •⁵ communicate result [1,5,6]	•³ $500=\frac{n}{2}(-6+4(n-1))$ •⁴ eg $4n^2-10n-1000=0$; 17.1 •⁵ 18	**3**

Notes:

1. Where a candidate adopts a non-algebraic approach, award 0/3.
2. At •³ accept $500<\frac{n}{2}(-6+4(n-1))$.
3. Evidence for •³ may appear later in the solution, eg by comparing an incorrectly simplified expression with 500.
4. Candidates are not required to give the negative root at •⁴
5. Where a candidate produces a whole number for n at •⁴, •⁵ is available only if they increase this number by one
6. •⁵ is available only for a single positive value derived from attempting to solve a quadratic equation in standard form. Do not accept a range of values.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
10.			•[1] interpret $\frac{dV}{dt}$ [1,2]	•[1] $\frac{dV}{dt} = 12$	4
			•[2] state relationship	•[2] $\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$ or equivalent	
			•[3] find $\frac{dV}{dr}$ [3]	•[3] $\frac{dV}{dr} = 15\ r^2\pi$	
			•[4] evaluate $\frac{dr}{dt}$ [2,4,5,6,7]	•[4] $\frac{dr}{dt} = \frac{1}{125\pi}$ mm per minute	

Notes:

1. Where a candidate uses a variable other than t (or T) for time (eg "min"), this must be explicitly defined. Otherwise, •[1] is not available.
2. Where a candidate converts to centimetres and seconds, $\frac{dV}{dt} = 2 \times 10^{-4}$ and $\frac{dr}{dt} = 4.2 \times 10^{-6}$ cm/s.
3. Where a candidate equates V to $\frac{dV}{dr}$, •[3] is not available.
4. Do not accept a negative answer at •[4].
5. At •[4], candidates must explicitly identify $\frac{dr}{dt}$.
6. Accept an answer rounded or truncated to at least two significant figures.
7. At •[4], units must be correct.

Commonly Observed Responses:

Using implicit differentiation

$\frac{dV}{dt} = 15\pi r^2 \times \frac{dr}{dt}$ award •[2] and •[3].

Question			Generic scheme	Illustrative scheme	Max mark
11.		A	•[1] give counterexample[1] and communicate [1,2,3]	•[1] eg $3^2 + 4^2 = 25$ which is not prime	**3**
		B	•[2] state form of two consecutive integers [4,5,6]	•[2] eg $k,\ k+1,\ k \in \mathbb{Z}$	
			•[3] show $k^2 + (k+1)^2$ is odd and communicate [7,8]	•[3] $2(k^2 + k) + 1$ which is odd	

Notes:

1. Where the answer relating to statement A contains incorrect information (before, between or after correct information) •[1] is not available.
2. Alternative communication for "not prime" includes "$25 = 5 \times 5$" or "25 is divisible by 5" or "25 is composite."
3. Accept "A is false" for communication at •[1].
4. For •[2], accept "k is an integer" or "let the integers be k …" but do not accept other source sets in words or symbols.
5. Do not accept eg $2k,\ 2k+1$ at •[2], unless the candidate also considers $2k-1,\ 2k$.
6. Where a candidate starts by equating eg $k^2 + (k+1)^2$ to $2m+1$, •[2] is available only if m is not defined as an integer at this stage.
7. For the award of •[3], the candidate must deal with $k,\ k+1$ or both pairs of $2k,\ 2k+1$ and $2k-1,\ 2k$.
8. Accept "B is true" for communication at •[3].

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
12.			•[1] identify $\bar{z}$	•[1] $x - iy$ stated or implied by •[2]	**5**
			•[2] substitute for $z, \bar{z}$	•[2] $(x+iy)^2 + 20(x-iy) - 156 = 0$	
			•[3] equate real or imaginary parts [1]	•[3] $x^2 - y^2 + 20x - 156 = 0$ or $2xy - 20y = 0$	
			•[4] solve imaginary part [2,3]	•[4] $x = 10$	
			•[5] use real part to find pair of solutions [2,3,4]	•[5] $z = 10 \pm 12i$	

Notes:

1. For •[3], accept $2ixy - 20iy = 0$.
2. For the award of •[4] or •[5], the values of x and y must be real.
3. Where, at •[4], a candidate obtains a value of y which leads to two values of x, •[5] is still available.
4. At •[5], accept $x = 10$, $y = \pm 12$.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
13.	(a)		•[1] state expression •[2] obtain values for A and B	•[1] $\frac{A}{x}+\frac{B}{x+1}$ •[2] $A = -2$ and $B = 2$	**2**

Notes:

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
13.	(b)		•3 integrate to find "$uv -$" 1,2 •4 differentiate to find "$\int u'v\,dx$" 1,2,3 •5 obtain full solution 1,2,4	•3 $\frac{1}{3}xe^{3x} - \ldots$ •4 $\ldots \int \frac{1}{3}e^{3x}dx$ •5 $\frac{1}{3}xe^{3x} - \frac{1}{9}e^{3x} + c$	**3**

Notes:

1. Where a candidate integrates both functions, award 0/3.
2. Where a candidate differentiates both functions, award 0/3, unless they have communicated the intention to integrate (see COR B).
3. Disregard the omission of dx at •4.
4. Disregard the omission of $+c$ at •5

Commonly Observed Responses:

COR A

Use of tabular method.

	D	I
$+$	x	e^{3x}
$-$	1	$\frac{1}{3}e^{3x}$
$+$		$\frac{1}{9}e^{3x}$

Award •3 for first three rows, and •4 for the final row. Headings may differ.

COR B

Candidate communicates that $v' = e^{3x}$, $v \quad 3e^{3x}$.

Do not award •3 but •4 and •5 are still available. In the case of •5, this may be as a result of integrating correctly the second time, or as a repeated error.

COR C

Candidate chooses to differentiate e^{3x} and integrate x.

Do not award •3 but •4 is available. For the award of •5, further applications will be needed to arrive at the correct solution.

Question			Generic scheme	Illustrative scheme	Max mark
13.	(c)		•6 state form of integrating factor 1,2,3	•6 $e^{\int \frac{-2}{x(x+1)}dx}$ stated or implied	**5**
			•7 substitute partial fractions into expression for integrating factor 1,3	•7 $e^{\int \frac{-2}{x}+\frac{2}{x+1}dx}$	
			•8 find integrating factor in simplified form 1	•8 $\frac{(x+1)^2}{x^2}$	
			•9 rewrite as integral equation 1,3,4,5,6,7	•9 $\frac{(x+1)^2 y}{x^2} = \int xe^{3x}dx$	
			•10 integrate RHS and rearrange 1,6,7,8,9	•10 $y = \frac{x^2}{(x+1)^2}\left(\frac{1}{3}xe^{3x} \quad \frac{1}{9}e^{3x} \quad c\right)$	

Notes:

1. Where no attempt is made to produce an integrating factor, award 0/5.
2. Where a candidate writes "$P(x) = \frac{-2}{x(x+1)}$" and then writes eg "$e^{\int P(x)}$", •6 may be awarded.
3. At •6, •7 and •9 disregard the omission of dx.
4. For candidates who produce an incorrect integrating factor, •9 may still be available.
5. Accept an unsimplified (or incorrectly simplified) integrand on the RHS for the award of •9.
6. Where incorrect simplification of the RHS occurs before or after •9, •10 is not available.
7. Where a candidate uses a constant integrating factor, •9 and •10 are not available.
8. At •10, +c must be used appropriately.
9. At •10, accept $y = \frac{x^2\left(\frac{1}{3}xe^{3x} - \frac{1}{9}e^{3x} + c\right)}{(x+1)^2}$, or equivalent.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
14.	(a)	(i)	•[1] state vectors [1]	•[1] $\begin{pmatrix}-1\\2\\-9\end{pmatrix}$, $\begin{pmatrix}2\\-1\\3\end{pmatrix}$	**1**

Notes:

1. Accept vectors written horizontally, eg $(-1,2,-9)$.

Commonly Observed Responses:

		(ii)	•[2] evidence of strategy for finding normal [1]	•[2] eg $\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\-1 & 2 & -9\\2 & -1 & 3\end{vmatrix}$	**3**
			•[3] calculate normal [2]	•[3] $\begin{pmatrix}1\\5\\1\end{pmatrix}$	
			•[4] obtain equation [3]	•[4] $x+5y+z=5$	

Notes:

1. Do not award •[2] where the position vectors of A, B or C are used, or if no strategy is evident.
2. Accept any multiple of $\mathbf{n}$ for •[3].
3. Accept an unsimplified equation at •[4].

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
14.	(b)		●[5] state parametric equations of line	●[5] $x = 1 + \lambda$ $y = -1 - \lambda$ $z = -1 + 4\lambda$	**3**
			●[6] substitute into LHS of plane equation [1,2]	●[6] $1+\lambda+5(-1-\lambda)-1+4\lambda$	
			●[7] conclusion [2,3,4]	●[7] $-5 \neq 5$ the equation is inconsistent so the line and plane do not intersect	

Notes:

1. Do not withhold ●[6] if a candidate includes the RHS of the plane equation.
2. Demonstrating that a particular point on the line does not lie on the plane gains a mark only as part of a solution exemplified in the COR below.
3. Where a candidate produces an incorrect plane equation or incorrect parametric equations, ●[7] is unavailable if the line intersects the plane but is available otherwise.
4. The minimum requirements for the award of ●[7] are eg
   a. "$-5 \neq 5$" **and** "do not intersect"
   b. "$-5 = 5$" **and** "inconsistent/not true" **and** "do not intersect"
   c. "$\lambda = \frac{10}{0}$, which is undefined" **and** "do not intersect".

Commonly Observed Response:

Alternative method for ●[5] and ●[6]:

$\begin{pmatrix}1\\5\\1\end{pmatrix} \bullet \begin{pmatrix}1\\-1\\4\end{pmatrix} = 0$, so the line is parallel to the plane award ●[5]

Substitute eg $(1,-1,-1)$ into LHS of plane equation award ●[6]

Question			Generic scheme	Illustrative scheme	Max mark
15.	(a)		•¹ separate variables and write down integral equation [1,2]	•¹ $\int \frac{dW}{36-W} = \int \frac{dt}{120}$	**5**
			•² integrate LHS [1,3]	•² $-\ln(36-W)$	
			•³ integrate RHS [1,4]	•³ $\frac{1}{120}t + c$	
			•⁴ find constant of integration [1,4,5,6]	•⁴ $c = -\ln 28$	
			•⁵ express W in terms of t [1,4,5,7,8,9]	•⁵ $W = 36 - 28e^{-\frac{1}{120}t}$	

Notes:

1. Where a candidate attempts to integrate an expression involving W with respect to t, award 0/5
2. .•¹ is not available if either $\int dW$ or $\int dt$ is omitted.
3. For the award of •², $-\ln\ldots$ must be present. Accept $-\ln|36-W|$.
4. •³ may be awarded if constant of integration is omitted. However, •⁴ and •⁵ are unavailable if no constant of integration subsequently appears.
5. If the constant of integration is not given as an exact value, award •⁴ only if the answer is correct to two significant figures (-3.3). However, do not award •⁵ if 28 does not appear in expression for W.
6. Where a candidate evaluates a constant after incorrectly rearranging, •⁴ is still available.
7. For the award of,•⁵ $-\ln\ldots$ must be present at •².
8. Accept equivalent answers- eg $W = 36 - \frac{28}{e^{\frac{1}{120}t}}$.
9. Do not award •⁵ for $W = 36 - e^{-\frac{1}{120}t+\ln 28}$ or $W = 36 - e^{\ln 28}e^{-\frac{1}{120}t}$.

Commonly Observed Responses:

Using integrating factor:

Integrating factor $= e^{\frac{1}{120}t}$ award •¹

$e^{\frac{1}{120}t}W = \int \frac{36}{120}e^{\frac{1}{120}t}$ award •²

$e^{\frac{1}{120}t}W = 36e^{\frac{1}{120}t} + c$ award •³

$c = -28$ award •⁴

$W = 36 - 28e^{-\frac{1}{120}t}$ award •⁵

Question			Generic scheme	Illustrative scheme	Max mark
15.	(b)		Method 1 •[6] find $\frac{dW}{dt}$ in terms of t •[7] evaluate $\frac{dW}{dt}$ [1,2] Method 2 •[6] evaluate W at $t=67$ and consider $\frac{dW}{dt}$ •[7] evaluate $\frac{dW}{dt}$ [1,2] Method 3 •[6] find $\frac{dW}{dt}$ •[7] evaluate $\frac{dW}{dt}$ [1,2]	Method 1 •[6] $\frac{dW}{dt}=\frac{36-(36-28e^{-\frac{1}{120}t})}{120}$ •[7] $\frac{7}{30}e^{-\frac{67}{120}}$ (kilograms per minute) Method 2 •[6] $W=20$ and $\frac{dW}{dt}=\ldots$ •[7] $0.13\ldots$ (kilograms per minute) Method 3 •[6] $\frac{dW}{dt}=\frac{28e^{-\frac{1}{120}t}}{120}$ •[7] $\frac{7}{30}e^{-\frac{67}{120}}$ (kilograms per minute)	**2**

Notes:

1. At •[7] accept any answer which rounds to 0.13 to two significant figures.
2. At •[7] units need not be given.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
15.	(c)		•[8] state limit and give justification [1,2,3,4,5,6,7]	•[8] $L = 36$ AND $e^{-\frac{1}{120}t} \to 0$ as $t \to \infty$ (or $28e^{-\frac{1}{120}t} \to 0$ as $t \to \infty$)	**1**

Notes:

1. There must be clear identification (eg "limit is 36", "36 as…", "… so 36", "…therefore 36") of the limit equalling 36 (or $35.\dot{9}$ but not $35.99\ldots$).
2. Where a candidate uses an expression which does not produce a limit, •[8] is not available.
3. Except as indicated in Note 4, do not accept $W = 36$.
4. Where a candidate proceeds from the original differential equation, accept $L = 36$ along with either:
   a. a statement that $\frac{dW}{dt} = 0$ (or a statement that the rate of change would be 0) "when" $W = 36$ (There must be the explicit appearance of $\frac{dW}{dt}$, $\frac{36-W}{120}$, or reference to derivative or rate of change. $W = 36$ may be implied by substitution into the original differential equation) or
   b. an explicit statement that $\frac{dW}{dt} \to 0$ (or rate of change/derivative…) as $W \to 36$.
5. Do not accept $e^{-\frac{1}{120}\infty}\ldots$, or $e^{-\frac{1}{120}t} = 0$ as part of a justification.
6. For consideration of an expression involving an exponential term, there must be reference (in words or symbols) to that term tending to zero as t increases.
7. Disregard incorrect information which does not relate to candidate's justification.

Commonly Observed Responses:

COR A

36 as $\frac{36-36}{120} = 0$, rate of change $= 0$… — award •[8]

COR B

36 as $\frac{36-36}{120} = 0$ — do not award •[8] (no mention of derivative)

COR C

$\frac{dW}{dt} = \frac{36-36}{120}$ 0 , therefore 36 — award •[8] (justification followed by conclusion)

COR D

$\frac{dW}{dt} = 0$ as $W \to 36$ so $L = 36$ — do not award •[8] (mixture of $=$, $\to$ in same statement)

COR E

$L = 36$ as, after this, $\frac{dW}{dt}$ is decreasing — do not award •[8]

COR F

36, as the rate of change would be 0 — do not award •[8] (no association with W)

[END OF MARKING INSTRUCTIONS]